



DESPECKLING OF MEDICAL IMAGES BASED ON GAUSSIAN MEMBERSHIP FUNCTION AND GRUNWALD-LETNIKNOV DERIVATIVE

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Abstract— Noise removal of Ultrasound (US), Computerized Tomography (CT) and Magnetic Resonance (MR) images are really challenging task in medical point of view. As the nature of the tumor, it can present any part of these images with any shape, size and contrast that creates the de-noising process further complicated. To overcome these issues, the proposed work presents an efficient Filter for image classification and de-noising based on Gaussian Membership Function and Grunwald-Derivative. The performance of the proposed filtering method is compared with Two Step Algorithm (TSA) which classifies the image region into three different regions and utilized suitable filters on these regions to partition the unwanted noise from the classified pixels. Simulation result shows the performance of proposed filtering process.

Keywords— Noise Removal, Ultrasound, Computer Tomography, Magnetic Resonance, Gaussian Membership, Grunwald-Derivative

I. INTRODUCTION

In image processing field, Researchers are frequently creating novel techniques to develop mathematical analysis for making them further accurate in order to decrease the negative health issues and cost effect. In 1978, [1] Shepp and Kruskal analyzed various reconstruction models in imaging systems by applying 'mathematical phantom' which involves section of body simulating that could be mathematically defined by a function. Medical images like US, CT and MR are very essential for health care and life sciences. Various innovations include in medical images are basically connected to the mathematical analysis (Jain, 2013)[2]. All techniques used for developing medical image processing are depends on exact mathematical methods, models and formulations. In addition, algorithm developed using mathematical analysis fulfill the purposes. Software depends these analysis provide the effective guidance for various image procedures such as non-invasive surgery planning, biopsy and radiation therapy. The presence of speckle noise in medical images is a significant problem in image processing. It corrupts the essential characteristics of

captured image and reduces the physician's understanding capability about the corrupted image. Hence, in medical image processing, de-noising or filtering has turned into a necessary preprocessing task. Speckle noise is known as multiplicative which follows gamma distribution. Various number of speckle filters have been presented in the review of literature.

[3] Filters depend on local statistics show good result for homogeneous areas but are unsuccessful in edge region. Most of the diffusion methods eliminate noise from medical image by employing PDE. [4] Perona and Malik presented a technique which stop diffusion throughout the edges. But this it is only suitable for additive noise. Generally, medical images consist of unrelated similar pixels dotted all over the image. Deseckling filter based on Non-local means replaces the image similar pixels with their average. But its efficiency decreases due to existence of speckle noise after filtering process. [5] For ultrasound denoising, a new bilateral filtering method has been proposed. A weighted speckle denoising algorithm is introduced in [6] for SAR images. This technique replaces every pixel with fuzzy rule. But it is unsuccessful to protect the image part which contains more texture and details involved. [7] Binaee proposed a procedure which classifies the image areas into several regions with image gradient and subsequently calculates non-local mean value of similar pixel windows. In addition to these, by applying butter worth filter, a denoising method for temporal echo images has been proposed by Parisa et al. [8]. Filter based on fuzzy logic for ultrasound images is a good contribution towards medical image processing however its performance is affected by high noise variance[9]. [10] Nageswari et al. discussed a two step methodology which categorizes an image into three different regions and utilized suitable filters to remove speckle noise present in the classified regions. In recent days, Fractional calculus plays an essential role in despeckling medical images. With L1 regularization terms and fractional differential operators, Yan et al. [11] introduced an edge preserving filter for image decomposition. For preprocessing of image fusion, Zhang et al. [12] used masks based on fractional-order differential. The image source has widened among the high and low frequency parts. This effectively preserves the edges as well as decreasing unwanted whole tone within the image.



[13, 14, 15] Few researchers combined theory of fractional order calculus and total variation for medical image processing. This combination is very effective for smoothing the image homogeneous regions.

II. PROPOSED METHOD

This section explains a filtering method for medical image classification and de-noising based on Gaussian Membership Function and Fractional Order Integration. Initially an image with noisy pixels is classified into various regions such as detail, homogeneous and edge by using Co-efficient of Variation and Gaussian Membership Function. Subsequently, to obtain an effective de-noising image, appropriate filters such as Mean for homogeneous region and Fractional Order Integration Filter for detail and edge regions is applied on the classified image pixels.

A. Noise Model

Let us consider the observed images be $H(x, y)$ with size $X \times Y$. It can be mathematically modeled as

$$H(x, y) = G(x, y) \cdot \beta(x, y) + \gamma(x, y) \quad (1)$$

where, $G(x, y)$ is noise free image. $\beta(x, y)$ and $\gamma(x, y)$ denotes the multiplicative and additive noise respectively. Image H has dimensions $X \times Y$. where, $x=1 \dots X$ and $y=1 \dots Y$. The effect of additive noise is very low when compared with multiplicative, so the equation (1) can be remodeled as

$$H(x, y) = G(x, y) \cdot \beta(x, y) \quad (2)$$

B. Image classification

Coefficient of Variation (COV) is defined as the ratio between Mean and Standard Deviations. It is used as image region classifier. It can be represented as

$$\text{Coefficient of variation} = \frac{\text{Standard Deviation}}{\text{Mean}}$$

A pixel that corresponds to homogeneous region has low COV values. Similarly, pixels with intermediate and high value represent the detail and edge region. On basis of this concept, noisy image pixels are partitioned into edge, homogeneous and detail regions. During image filtering, it is not easy to remove noise, as it widens all over the image pixels. Hence, for describing fuzzy values for the pixels in classified regions, a membership function called Gaussian is employed. The main advantages for employing Gaussian Function are it is symmetric about its mean and tends to zero only after some standard deviations. Hence, by using Gaussian Membership Function, the value of COV calculated for the image noisy pixels is mapped to the fuzzy domain. It can be defined as

$$\mu_g^k(u) = e^{-\frac{(u-m_k)^2}{2\sigma_k^2}}, \quad k = 1, 2, 3$$

where, μ_g^k represents the set of fuzzy values of the pixels $N \times N$. m_k and σ_k^2 denotes the mean and variance of $N \times N$ pixels. Here $N \times N$ represents a square window. To define the three classes such as homogeneous, edge and detail, threshold values p , q , and r are evaluated by using the equation given below.

$$H(x, y) \in \begin{cases} \mu_g^k = \text{edge} & u > q \\ \mu_g^k = \text{detail} & u \geq p \text{ and } u \leq r \\ \mu_g^k = \text{homogeneous} & u < q \end{cases}$$

$$\begin{aligned} \text{Where, } p &= \text{Maximum of COV } [H(x, y)]_{X \times Y} \\ r &= \text{Maximum of COV } [\text{grad } H(x, y)]_{X \times Y} \\ q &= \text{Average}[a, c] \end{aligned}$$

Here COV represents the co-efficient of variation of $H(x, y)$. Pixels that correspond to homogeneous region have low COV values. To define intermediate and detail region, a threshold 'p' is needed. To define 'p', a value with maximum COV computed for the image $H(x, y)$ has chosen. Generally, edge pixels have maximum value of COV. Hence the threshold 'r' is need to be evaluated. Since gradient operation is efficient in edge separation, the gradient value of $H(x, y)$ is found for which COV is evaluated. Then the threshold 'r' is calculated as maximum of COV. The average value of 'p' and 'r' is considered as a threshold 'q'. Hence depending on noise ratio added to the image, thresholds p , q and r varies adaptively, and noisy input pixel is classified accordingly.

C Speckle De-noising Filters

During noise detection process, based on COV, every noisy pixel is mapped into Gaussian membership function. Every pixel belongs to fuzzy domain is characterized and analyzed into three regions by this detection process. Appropriate filters are required to eliminate the noise with no affect for image structural details.

Homogenous region

Since homogeneous part has low effected components like edge and detail region, we will reconstruct homogeneous region noisy pixels with the average value of 3×3 window pixels around it. The equation that can be used to process the classified noisy homogeneous pixels is given by

$$D(x, y) = \frac{1}{9} \sum_{i=-1}^1 \sum_{j=-1}^1 H(x+i, y+j)$$

Where $D(x, y)$ is the de-noised image

Detail and edge Region

Average Filter is not suitable for filtering detail and edge region noisy pixels. Because we have a chance to loss structural details of the image during filtering process. Hence Fractional Order Integration filter has selected for treating



detail and edge region noisy pixels. It has the ability to eliminate noise whereas preserving edge and fine details. For

any arbitrary square integrable signal $S(t) \in L^2(R)$, its v order differential is represented as

$$D^v S(t) = \frac{d^v S(t)}{dt^v}$$

From the theory of signal processing, its Fourier Transform is described as

$$\begin{aligned} D_v S(t) &\stackrel{FT}{\longleftrightarrow} (\widehat{D}^v f)(w) = (iw)^v \widehat{f}(w) \\ &= |w|^v \exp[i\theta^v(w)] \widehat{f}(w) \\ &= |w|^v \exp\left[\frac{\alpha\pi}{2} \text{sgn}(w)\right] \widehat{f}(w), v \in R \end{aligned}$$

Where, D_v represents the differential operator with order v , W denotes angular frequency,

$$(iw)^v = |w|^v \exp\left[\frac{\alpha\pi i}{2} \text{sgn}(w)\right]$$

is the filter function of fractional calculus filter and $\text{sign}(\cdot)$ denotes the numeric symbol of the integral part.

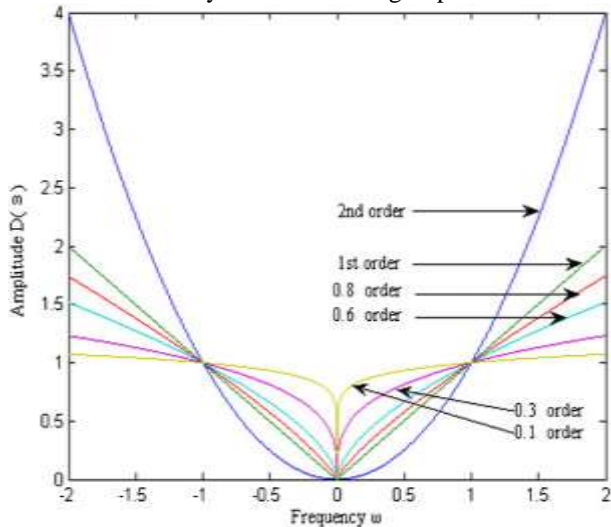


Fig. 1. Amplitude–frequency curve of differential

From the figure, we come to know that, the amplitude value can be enlarged by fractional operator for low frequency part ($0 < w < 1$), which represents the image smooth regions. However, for intermediate and high frequency part ($w > 1$), the fractional operator reacts an attenuation function and the attenuation amplitude will become stronger as the fractional order increasing. By considering this feature of fractional calculus, using Gamma function, it can be enlarged from integer to fraction order. This feature reveals that the fractional integral operator can develop the low frequency signal while attenuating the high frequency signal, and has some effect on the noised images. Figure1 shows the frequency amplitude characteristic curves with various fractional orders of We know that the Fractional Calculus formula with $-$ order of Grunwald – Letnikov is.

$$S^v(t) = \frac{d^v s}{dt^v} = \lim_{h \rightarrow 0} \left(\frac{1}{h}\right)^v \sum_{r=0}^v (-1)^r \frac{\Gamma v+1}{\Gamma r+1 \Gamma v-r+1} s(t-rh)$$

We know that, for two dimensional images, the smallest distance of gray intensity varies among its two adjacent pixels. Hence the time duration of a two-dimensional image along x and y axis can only be calculated in the pixel unit. Thus $h=1$, then. Therefore, according to G-L fractional calculus expression, we can get the differential expression with of order of fractional differential $s(t)$ is

$$\frac{d^v S(t)}{dt^v} \approx S(t) + (-v)S(t-1) + \frac{(-v)(-v+1)}{2} S(t-2) + \dots + \frac{\Gamma v+1}{r! \Gamma v-r+1} S(t-r).$$

For any function $S(x, y) \in L^2(R^2)$, the differential expressions for the partial calculus with ‘ v ’ order with respect to x and y is as follows:

$$\frac{d^v S(x, y)}{dx^v} \approx S(x, y) + (-v)S(x-1, y) + \frac{(-v)(-v+1)}{2} S(x-2, y) + \dots + \frac{\Gamma v+1}{r! \Gamma v-r+1} S(x-r, y)$$

$$\frac{d^v S(x, y)}{dy^v} \approx S(x, y) + (-v)S(x, y-1) + \frac{(-v)(-v+1)}{2} S(x, y-2) + \dots + \frac{\Gamma v+1}{r! \Gamma v-r+1} S(x, y-r)$$

Using the above two formulas, the mask is attained by overlaying the partial fractional integral in eight directions as shown in Figure 2, hence it is rotation invariant.

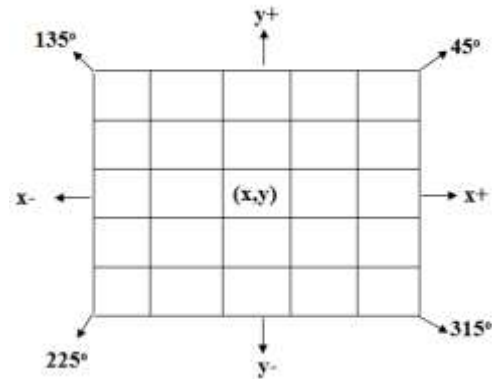


Fig. 2. Partial fractional integral in eight directions

From the figure of frequency amplitude characteristic curves of the Fractional Order Integral, we come to know that, it has different responses at different frequencies. Hence, the edge and detail region pixels are denoised by utilizing the following equation:

$$D(x, y) = H(x, y) * \text{mask}$$

For detail region, order α is utilized because there are no heavy frequency components to be protected and hence greatest quantity of noise would be mitigated from detail region. Similarly, for edge region, order β is utilized because it less mitigates the heavy frequency components like edges and



hence tiny texture of edge region will be preserved. Utilizing H, convolve masks for x and y direction one by one and then calculate mean to obtain the resulting image D. Figure 3 shows the flow diagram of the proposed filtering method.

III. EXPERIMENT AND RESULT

A. Steps involved in the proposed method

1. Let $H(x, y)$ be the input image with size $X \times Y$.
2. Calculate COV for each pixel of $H(x, y)$.

$$\text{Coefficient of Variation} = \frac{\text{Standard Deviation}}{\text{Mean}}$$

3. Calculate thresholds for Gaussian Function using
 $p = \text{Max COV}[H(x, y)]_{X \times Y}$
 $q = \text{Avg}[p, r]$

$$r = \text{COV}[\text{grad } H(x, y)]_{X \times Y}$$

4. Separate each pixel of $H(x, y)$ into different three regions using Gaussian Function

$$H(x, y) \in \begin{cases} \mu_g^k = \text{edge} & u > q \\ \mu_g^k = \text{detail} & u \geq p \text{ and } u \leq r \\ \mu_g^k = \text{homogeneous} & u < q \end{cases}$$

5. Employ appropriate filter to denoise every pixels in each region

$$D(x, y) = \begin{cases} \eta_1 & H(x, y) \in \text{edge} \\ \eta_2 & H(x, y) \in \text{detail} \\ \eta_3 & H(x, y) \in \text{homogeneous} \end{cases}$$

Where $\eta_1 = \frac{1}{9} \sum_{j=-1}^1 \sum_{k=-1}^1 H(x+j, y+k)$

$$\eta_2 = H(x, y) * \text{mask with } v = 0.7$$

$$\eta_3 = H(x, y) * \text{mask with } v = 0.5$$

6. Obtain resultant image $D(x, y)$ as the de-noised image.

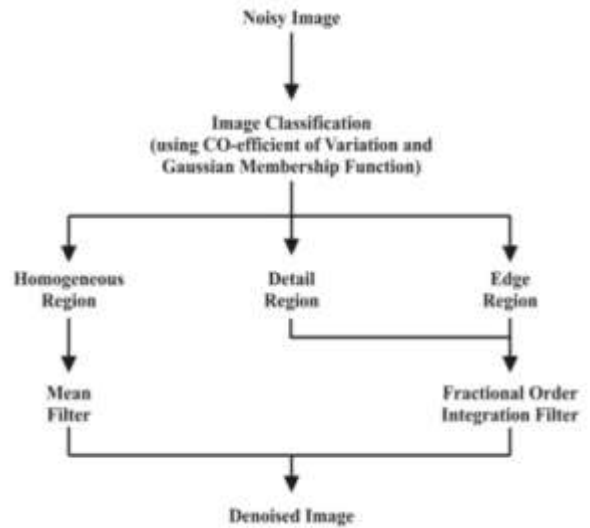


Fig. 3. Flow diagram of the proposed HFCF method

B. Numerical Results

Speckle noise is added to the Ultrasound image, Magnetic Resonance image and Computerized Tomography image. The test images were corrupted with noise levels varying from 10% (low noise ratio) to 90% (extreme high noise ratio). The numerical results such as MSE, PSNR AND SSIM are compared between existing TSA [10] and proposed method for various medical images. It can be shown in the tables from 4.1 to 4.3. It is clear that the proposed filter performs well for Magnetic Resonance Image, Computerized Tomography and Ultrasound test images which are corrupted at different noise ratios.

Table -1 Performance of MSE of the Existing TSA and Proposed Filter for various test images

Images & Filters	MR image		CT image		US image	
	TSA	Proposed	TSA	Proposed	TSA	Proposed
Noise Level (%)						
10	0.000417	0.000303	0.0017	0.0014	0.000910	0.000356
20	0.000418	0.000320	0.0020	0.0018	0.000918	0.00041
30	0.000428	0.000339	0.0024	0.0022	0.000920	0.00043
40	0.000440	0.000353	0.0030	0.0027	0.000934	0.00048
50	0.000459	0.000379	0.0035	0.0032	0.000942	0.00052
60	0.000478	0.000402	0.0037	0.0034	0.000955	0.00057
70	0.000496	0.000438	0.0045	0.0041	0.000968	0.00061
80	0.000525	0.000475	0.0051	0.0049	0.000971	0.00065
90	0.000555	0.000537	0.0061	0.0057	0.000990	0.00070



Table:2 Performance of PSNR of the Existing TSA and Proposed Filter for various test images

Images & Filters	MR image		CT image		US image	
	TSA	Proposed	TSA	Proposed	TSA	Proposed
Noise Level (%)						
10	0.000417	0.000303	0.0017	0.0014	0.000910	0.000356
20	0.000418	0.000320	0.0020	0.0018	0.000918	0.00041
30	0.000428	0.000339	0.0024	0.0022	0.000920	0.00043
40	0.000440	0.000353	0.0030	0.0027	0.000934	0.00048
50	0.000459	0.000379	0.0035	0.0032	0.000942	0.00052
60	0.000478	0.000402	0.0037	0.0034	0.000955	0.00057
70	0.000496	0.000438	0.0045	0.0041	0.000968	0.00061
80	0.000525	0.000475	0.0051	0.0049	0.000971	0.00065
90	0.000555	0.000537	0.0061	0.0057	0.000990	0.00070

Table:2 Performance of SSIM of the Existing TSA and Proposed Filter for various test images

Images & Filters	MR image		CT image		US image	
	TSA	Proposed	TSA	Proposed	TSA	Proposed
Noise Level (%)						
10	0.961	0.965	0.846	0.861	0.800	0.812
20	0.958	0.964	0.836	0.844	0.780	0.796
30	0.956	0.962	0.828	0.832	0.760	0.772
40	0.955	0.960	0.812	0.813	0.756	0.765
50	0.953	0.957	0.792	0.802	0.742	0.753
60	0.951	0.954	0.766	0.787	0.739	0.742
70	0.948	0.949	0.737	0.774	0.730	0.737
80	0.946	0.944	0.703	0.753	0.723	0.731
90	0.936	0.939	0.667	0.727	0.800	0.812

The visual performance of the existing TSA and proposed Filter for various standard test images such as MRI, CT and US is demonstrated as follows:

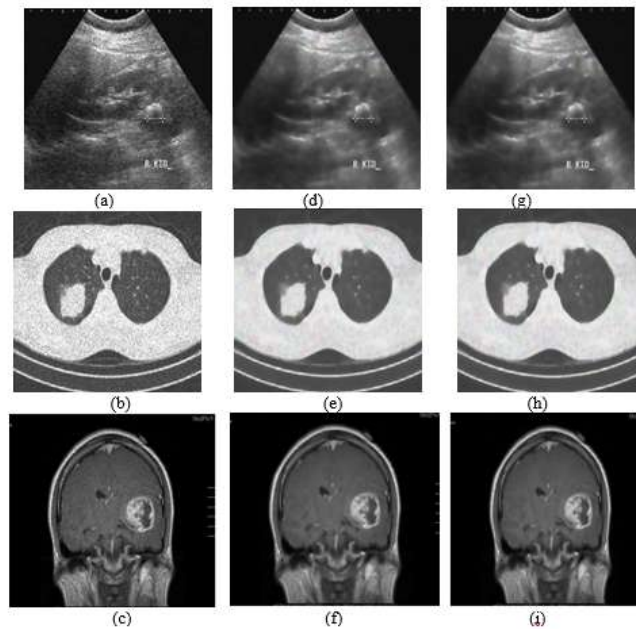


Fig. 4. Visual Representation of Existing MFBF and Proposed HFCF for MRI, CT and US images.

The above figure illustrates the visual demonstration of the existing MFBF and proposed HFCF for various test images such as MRI, CT and US. Figure 4 (a), (b) and (c) displays an image part corrupted with 40% speckle noise. Figure 4 (d), (e), (f), and (g), (h), (i) shows the noise filtered images with existing TSA and proposed filter respectively. It is clear that the proposed filter presented better visual effect than existing TSA while suppressing the speckle noise.

IV. CONCLUSION

A Hybrid Filter based on Coefficient of Variation, Gaussian Membership Function and Grunwald –Letnikov is presented in this paper. Initially, the input images are classified into three different regions such as homogeneous, detail and edge by using Coefficient of Variation and Gaussian Membership Function. Then the appropriate filters like Mean for homogeneous region and Grunwald –Letnikov for detail and edge region are applied to denoising the classified images. The proposed filter performed very well during image classification and in noise suppression or removal. It not only removes noise but also preserves edges and other important image particulars.

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